94 #1

(a) $f' = 12x^3 + 3x^2 - 42x$

$m = 12(2)^3 + 3(2)^2 - 42(2)$

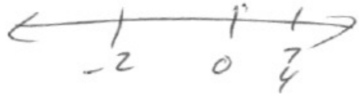
$m = 24$

$24 = \frac{y+28}{x-2}$

(b) $f' = 12x^3 + 3x^2 - 42x$

$0 = 3x(4x^2 + x - 14)$

$0 = 3x(4x - 7)(x+2)$



check each pt

$f(-2) = -44$ $\qquad f(\frac{7}{4}) = -30.8$

(c) $f'' = 36x^2 + 6x - 42$

$= 6x + x - 7$

$= (6x+7)(x-1)$

Pts of inflection

1994

#2

ⓐ $\int_0^4 (e^x - x)\,dx$

~~$(e^4 - 4) - e^0 - 0 = e^4 - 4 - 1$~~

~~$= e^4 - 5$~~

$e^x - \frac{x^2}{2}\Big]_0^4 = e^4 - \frac{16}{2} - (e^0 - 0)$

$= e^4 - 8 - 1 = e^4 - 9$

ⓑ

~~$\int_0^4 \pi (e^x - x)^2\,dx$~~

$\pi \int_0^4 (e^x)^2 - (x)^2\,dx$

$\pi \int_0^4 e^{2x} - x^2$

$\pi \left(\frac{1}{2}e^{2x} - \frac{x^3}{3}\right]_0^4 = \pi\left(\frac{1}{2}e^8 - \frac{4^3}{3}\right) - \pi\left(\frac{1}{2}e^0 - 0\right)$

$= \pi\left(\frac{1}{2}e^8 - \frac{64}{3}\right) - \frac{1}{2}\pi$

ⓒ $2\pi \int_0^4 x(e^x - x)\,dx$

1994

3

(a) $2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$

$$(x+2y)\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

(b) x-intercepts

let $y=0$

$$\cancel{2x^2 + x(0) + 0^2 = 27}$$

$$2x = 27$$

$$x = \frac{27}{2}$$

$$x^2 + x(0) + (0)^2 = 27$$

$$x^2 = 27$$

$$x = \pm\sqrt{27}$$

$$m = \frac{-2(\sqrt{27}) - 0}{\sqrt{27}} = -2$$

$$m = \frac{-2(-\sqrt{27})}{-\sqrt{27}} = -2 \therefore \text{ Parallel}$$

(c)

Horiz.

Vertical where undefined

$$x + 2y = 0$$

$$\cancel{2y = x} \quad x = -2y$$

$$\cancel{y = \frac{x}{2}}$$

$$x^2 + xy + y^2 = 27$$

$$(-2y)^2 + (-2y)y + y^2 = 27$$

$$4y^2 - 2y^2 + y^2 = 27$$

$$3y^2 = 27$$

$$y = \pm 3$$

Pts $(-6,3)$ $(6,-3)$

1994
#4

$v = t\ln t - t$ at $t=1$ position $x(1)=6$

a) $a = v' = t\frac{1}{t} + (\ln t) - 1$
$= 1 + \ln t - 1 = \ln t$

b) $t\ln t - t > 0$
$t(\ln t - 1) > 0$

		0		e	
t	$-$	0	$+$		$+$
$\ln t - 1$	$-$	$-$			$+$
	$+$	0	$-$	0	$+$

$\ln t - 1 = 0$
$\ln t = 1$
$\log_e t = 1$
$t = e$

$t > 0$ by original problem

$t > e$

c) check critical pts

take first derivative

$v' = \ln t$

$\ln t > 0$

$\log_e t = 0$
$t = 1$

minimum velocity occurs at $t=1$

$v(1) = 1\ln 1 - 1 = -1$

d) ~~$\int t\ln t - t$~~ by

next pg)

1994

#4 d)

$$\int (t\ln t - t)\,dt$$

$\int u\,dv$

$u = \ln t \quad dv = t$

$du = \frac{1}{t} \quad v = \frac{1}{2}t^2$

$uv - \int v\,du$

$\frac{1}{2}t$

$$\frac{1}{2}(\ln t)t^2 - \int \frac{1}{2}t^2(\cancel{t})\frac{1}{t}$$

$$= \frac{t^2}{2}\ln t - \frac{t^2}{4} - \frac{t^2}{2} + c$$

$$= \frac{t^2}{2}\ln t - \frac{3}{4}t^2 + c$$

$x(1) = 6$

$$6 = \frac{1}{2}\ln 1 - \frac{3}{4}(1)^2 + c$$

$$6 = \frac{1}{2}(0) - \frac{3}{4} + c$$

$$c = \frac{27}{4}$$

$$x(t) = \frac{1}{2}t^2\ln t - \frac{3}{4}t^2 + \frac{27}{4}$$

94

AB
#5

(a) $\frac{dc}{dt} = 6$ " per sec

$\frac{dp}{dt} = ?$

$C = 2\pi r$ $\quad$ $A = \pi r^2$ $\quad$ $P = 8r$

$\frac{dc}{dt} = 2\pi \frac{dr}{dt}$

$6 = 2\pi \frac{dr}{dt}$

$\frac{6}{2\pi} = \frac{dr}{dt}$

$\frac{3}{\pi} = \frac{dr}{dt}$

$P = 8r$

$\downarrow$

$\frac{dp}{dt} = 8\frac{dr}{dt}$

$\frac{dp}{dt} = 8\left(\frac{3}{\pi}\right)$

$= \frac{24}{\pi}$ " per sec

(b)

$A = 25\pi$

~~$\frac{dA}{dt} = ?$~~

Area between $= (2r)^2 - \pi r^2$

$A = 4r^2 - \pi r^2$

$\frac{dA}{dt} = 8r\frac{dr}{dt} - 2\pi r\frac{dr}{dt}$

$= (8r - 2\pi r)\frac{dr}{dt}$

$Area_c =$

$25\pi = \pi r^2$

$\underline{r = 5}$ $\rightarrow$ also $\frac{dr}{dt} = \frac{3}{\pi}$

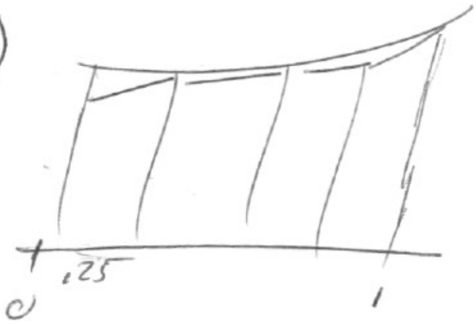
$\frac{dA}{dt} = (8r - 2\pi r)\frac{dr}{dt}$

$= (8(5) - 2\pi(5))\frac{3}{\pi}$

$= (40 - 10\pi)\frac{3}{\pi}$ in^2/sec

Q4

#6 a)



$$.25\left(\frac{\sin 0^2+\sin(.25)^2}{2}\right)+.25\left(\frac{\sin(.25)^2+\sin(.5)^2}{2}\right)$$

$$+\ .25\left(\frac{\sin(.5)^2+\sin(.75)^2}{2}\right)+.25\left(\frac{\sin(.75)^2+\sin 1^2}{2}\right)$$

≈ 0.316

b) $\cancel{F' = \cos(t)^2}$

$F(x)=\int_0^x \sin(t^2)\,dt$

$\therefore$ First derivative $= \sin(t^2)$

$\sin(t^2)=0$

$x^2 = 0, \pi, 2\pi, \ldots$

$\sqrt{x^2}=x=0, \sqrt{\pi}, \sqrt{2\pi}$

$x-\sqrt{\pi}$ $-$ 0 + +

$x-\sqrt{2\pi}$ $-$ $-$ 0 +

0 + $\sqrt{\pi}$ $-$ $\sqrt{2\pi}$ + 3

Increasing $[0,\sqrt{\pi}]$, + $[\sqrt{2\pi}, 3]$

c)

$$k = \cancel{\frac{F(3)-F(1)}{3-1}}\ \frac{\int_1^3 \sin(t^2)\,dt}{3-1}$$

$\cancel{2k = F(3)-F(1)}$

$2k = F(3)-F(1)$

Integral?

sub

$u=\sin(t^2)$ $dv = dt$

$du = \cos(t^2)2t$ $v=t$